08:38:52 OCA PAD AMENDMENT - PROJECT HEADER INFORMATION 06/11/92

Active

Project #: E-16-603 Cost share #: E-16-341 Rev #: 9
Center # : R6662-0A0 Center shr #: F6662-0A0 OCA file #:
Work type : RES
Contract#: DAAL03-89-K-0007 Mod #: P00004 Document : CONT
Prime #: Contract entity: GTRC

Subprojects ? : N CFDA: N/A
Main project #: PE #: 611102

Project unit: AERO ENGR Unit code: 02.010.110
Project director(s):
HODGES D H AERO ENGR (404)894-8201

Sponsor/division names: ARMY / ARO, RES TRIANGLE PARK, NC
Sponsor/division codes: 102 / 001

Award period: 881201 to 920930 (performance) 921130 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	277,075.00
Funded	0.00	277,075.00
Cost sharing amount		23,230.00

Does subcontracting plan apply ?: N

Title: STABILITY OF ELASTICALLY TAILORED ROTOR SYSTEMS

PROJECT ADMINISTRATION DATA

OCA contact: Ina R. Lashley 894-4820

Sponsor technical contact	Sponsor issuing office
DR GARY L ANDERSON, ENG SCI DIV	MS PATSY S ASHE, C.O.
(000)000-0000	(919)549-0641
US ARMY RESEARCH OFFICE	US ARMY RESEARCH OFFICE
P O BOX 12211	P O BOX 12211
RESEARCH TRIANGLE PARK NC 27709-2211	RESEARCH TRIANGLE PARK NC 27709-2211

Security class (U,C,S,TS) : U ONR resident rep. is ACO (Y/N): Y
Defense priority rating : N/A supplemental sheet
Equipment title vests with: Sponsor GIT X
NONE PROPOSED. SEE BLOCK 27 (CONT'D SHEET) RE PRIOR APPROVALS.
Administrative comments -
P00004 AUTHORIZES A 3-MONTHS NO-COST EXTENSION, AS REQUESTED 5/21/92.

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 12/14/92

Project No. E-16-603 Center No. R6662-0A0

Project Director HODGES D H School/Lab AERO ENGR

Sponsor ARMY/ARO, RES TRIANGLE PARK, NC

Contract/Grant No. DAAL03-89-K-0007 Contract Entity GTRC

Prime Contract No.

Title STABILITY OF ELASTICALLY TAILORED ROTOR SYSTEMS

Effective Completion Date 920930 (Performance) 921130 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	
Final Report of Inventions and/or Subcontracts	Y	
Government Property Inventory & Related Certificate	Y	
Classified Material Certificate	N	
Release and Assignment	Y	
Other	N	

CommentsEFFECTIVE DATE 12-1-88. CONTRACT VALUE $277,075

Subproject Under Main Project No.

Continues Project No.

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Managment	Y
Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
Project File	Y
Other HARRY VANN-FMD	Y
FRED CAIN-OOD	Y

NOTE: Final Patent Questionnaire sent to PDPI.

E-16-603

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 December 1988 – 30 May 1989

3. TITLE OF PROPOSAL: Stability of Elastically Tailored Rotor Systems

4. CONTACT OR GRANT NUMBER: DAAL03-89-K-0007

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES: None (Project began this period)

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: Dewey H. Hodges and Lawrence W. Rehfield[1], principal investigators, and Mark V. Fulton, Graduate Research Assistant

BRIEF OUTLINE OF RESEARCH FINDINGS

A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to "tailor" the properties of composites to meet specific design requirements. Design of structures using material and geometric tailoring is an old subject and has received widespread attention during the last decade with the advent of composite materials. One class of problems for which tailoring of composite material properties has received relatively little attention is in the area of rotorcraft design.

It is well known that kinematical couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize these couplings from composite material properties in rotorcraft design must include a concomitant investigation of the influence of the coupling on stability. Second, there remains the possibility of tailoring the rotor blade structure and improving stability to the point that auxiliary mechanical dampers become unnecessary. Another possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. The principle is that the blade rotational speed, through extension-twist coupling, can be used to alter the twist of the blade to favorably influence the aerodynamics. These and other areas have strong potential for breakthroughs in design methodology. This research project concerns the influence of elastic couplings on the aeroelastic stability of rotor systems.

The project has been underway for about half a year. During this time the effort has concentrated on the development of a simple analysis that will predict the frequencies and mode shapes of uniform composite blades. We have set out to limit ourselves to uniform blades for the purpose of the simplified aeroelastic stability analysis to which the present work will ultimately lead. A more complex analysis, to be developed later, will have the capability to handle nonuniformities. What

[1]Prof. L. W. Rehfield is with the University of California at Davis. His effort in this project is supported under subcontract.

has been shown so far is that this simplified analysis may be efficiently performed as a two-stage operation. The first step is to analyze a cross section (a two-dimensional problem). The second step is a global bending-torsion-extension-shear analysis (a one-dimensional problem).

First Step: Two-Dimensional Analysis:

We have established theoretical proofs that the linear two-dimensional analysis can be undertaken independently of the deformation within some limits, of course. This was done under ARO sponsorship through the Center of Excellence for Rotary Wing Aircraft (CERWAT) grant. Because of the nature of the two-dimensional analysis required, it was not necessary that we develop our own since existing methods are adequate for the purposes of this project. We have two codes which perform the first step operation in differing degrees of accuracy. The first is a version of Dr. Marco Borri's code developed in Italy and adapted for our use by Georgia Tech personnel; this code is called NABSA (Nonhomogenous, Anisotropic Beam Slice Analysis). The second is written by Mr. Mark Nixon at the U.S. Army Aerostructures Directorate, Langley Research Center. It is restricted to thin-walled multi-celled beams. The results of the first step are (a) a six-by-six stiffness matrix for the cross section; (b) the warping distribution over the cross section; and (c) the stress distribution over the cross section. The only part of results from the first step that we really need for the second step is the stiffness matrix.

Second Step: One-Dimensional Analysis

The results for the second step are the global structural dynamics of the blade (such as frequencies and mode shapes). The second step analysis, developed under this grant, does not care where the six-by-six stiffness matrix comes from. There are several ways to accomplish this second step. The first approach considered was a transfer matrix method. This method was selected because of its accuracy and its ease of implementation. However, it is well known that one cannot guarantee that all the eigenvalues can be extracted over a given range with this method. This can be a problem when dealing with systems with highly coupled modes such as composite beams. However, we needed something that could be developed quickly. After its development, we noticed that it was less efficient than we wanted for the simplified analysis, and there were problems controlling which roots were found. Although we supplied our subcontractor (Dr. Rehfield at the University of California, Davis) with a version of this code, and he is using it to study the effects of composite couplings on the frequencies and mode shapes of thin-walled composite beams, we are still pursuing a better way. This better way will also form a foundation on which to build the rotor blade stability analysis.

The better way is a simple and elegant finite element method developed by one of the principal investigators under the short-term innovative research program (STIRP) and extended under CERWAT, both funded by ARO. This method has been successfully applied in our other research programs for dynamics of rigid bodies, statics of beams, and optimal control of launch system trajectories. In accordance with [1], a very crude shape function can be used for all unknowns, and the forces and moments in the beam converge to the exact solution even more rapidly than the displacements and rotations. This method is being developed for linear structural dynamics of beams for use in the present study. Preliminary results indicate that the method will be more efficient than the transfer matrix method. It does give all the eigenvalues within some range, and it allows for simple treatment of beams with variable geometry.

Reference:

Hodges, Dewey H.: A Mixed Variational Formulation Based on Exact Intrinsic Equations for Dynamics of Moving Beams. *International Journal of Solids and Structures*, submitted for publication, 1989.

E-16-603

MEMORANDUM OF TRANSMITTAL

U. S. Army Research Office
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SUBMITTED FOR PUBLICATION TO (applicable only if report is manuscript):________

Sincerely,

Dewey H. Hodges

25327-EG

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 June – 30 November 1989

3. TITLE OF PROPOSAL: Stability of Elastically Tailored Rotor Systems

4. CONTACT OR GRANT NUMBER: DAAL03-89-K-0007

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges and Lawrence W. Rehfield[1]

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   Hodges, Dewey H.; Atilgan, Ali R.; Fulton, Mark V.; and Rehfield, Lawrence W.: Dynamic Characteristics of Composite Beam Structures. *Proceedings of the American Helicopter Society National Specialists' Meeting on Rotorcraft Dynamics*, Arlington, Texas, Nov. 13 – 14, 1989. (Submitted to the *Journal of the American Helicopter Society*.)

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: Dewey H. Hodges and Lawrence W. Rehfield[1], principal investigators; Mark V. Fulton and Stephen Chang[1], Graduate Research Assistants

BRIEF OUTLINE OF RESEARCH FINDINGS

A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to "tailor" the properties of composites to meet specific design requirements. Design of structures using material and geometric tailoring is an old subject and has received widespread attention during the last decade with the advent of composite materials. One class of problems for which tailoring of composite material properties has received relatively little attention is in the area of rotor blade design. The primary issue here is the overall suitability of tailored blades for practical applications. Blades must satisfy strength, weight, frequency, and lifetime requirements for structural stability; in addition, aeroelastic instabilities must lie outside the flight envelope.

It is well known that kinematical couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize the couplings from the use of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. There also remains the possibility of tailoring the rotor blade structure and improving stability to the point that auxiliary mechanical dampers become unnecessary. Another possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. The principle is that the blade rotational speed, through extension-twist coupling, can be used to alter the twist of the blade to favorably influence the aerodynamics. These and other areas have strong potential for breakthroughs in design methodology.

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[1]Prof. L. W. Rehfield is with the University of California at Davis. His effort in this project is supported under subcontract. Stephen Chang is a Graduate Research Assistant who works with him.

The project has been underway for about a year. An important subtask in order to proceed in this work is to create baseline configurations from which the payoffs due to aeroelastic tailoring can be clearly understood. A survey of present design methodology among the helicopter manufacturers, which will lead to a baseline definition, is nearing completion. During this time our analytical effort has concentrated on the development of simple analyses that will predict the frequencies and mode shapes of uniform composite blades. We have set out to limit ourselves to uniform blades for the purpose of the simplified aeroelastic stability analysis to which the present work will ultimately lead. In fact, the analysis developed for the study of dynamic characteristics has the capability to handle nonuniformities.

The dynamic analysis may be efficiently performed as a two-stage operation. The first step is to determine the cross sectional stiffnesses by means of either a simple analytical technique or by a finite element method (a two-dimensional problem). The second step is a global bending-torsion-extension-shear analysis (a one-dimensional problem). The only part of results from the first step that we really need for the second step is the stiffness matrix. Our work in this area is described in the last progress report.

Analysis of Dynamic Characteristics

The results for the second step include the global dynamic behavior of the blade (such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics). Two approaches were considered for calculation of the modes and frequencies. The first is a transfer matrix method also described in our last progress report. A second approach was also developed, which is described in the attached paper – namely a simple and elegant finite element method. This method has been successfully applied in our other research programs for dynamics of rigid bodies, statics of beams, and optimal control problems. In this method, a very crude shape function can be used for all unknowns, and the forces and moments in the beam converge to the exact solution even more rapidly than the displacements and rotations. This method is more efficient than the transfer matrix method, and it gives all the eigenvalues within a range which depends on the number of elements and the mode of interest. Furthermore, the method allows for simple treatment of beams with variable geometry.

Results we have obtained show the potentially *large* effects of including bending-shear coupling in the calculation of both static deflections [1] and dynamic characteristics. Modes associated with transverse displacements in a blade designed for extension-twist coupling will have a much lower frequency than they would if the bending-shear coupling *inherent* in such designs were not considered. The importance of this coupling is evident in static results which can be off by a factor of two if the coupling is not properly accounted for; *this error is independent of slenderness!* We further note that transverse shear strain must be in the analysis in order to obtain the correct stiffness model.

We also obtained results for composite beams in which the presence of the extension-twist coupling brings the frequencies of modes dominated by extensional displacement down around those of other modes such as those dominated by torsional and bending deformations. From these and other results, we conclude that knowledge of the complete 6-by-6 matrix of elastic constants is necessary for dynamics and stability analyses of composite rotor blades. Furthermore, the extensional displacement and strain must be in the model. This not only implies that previously developed rotor blade stability analyses do not contain the right ingredients for composite blade analysis, but that a significant portion of the results that have been published for composite blade stability is missing these important coupling effects.

Reference

1. Rehfield, Lawrence W.; Atilgan, Ali R.; and Hodges, Dewey H.: Some Considerations on the Nonclassical Behavior of Thin-Walled Composite Beams. *J. American Helicopter Society*, to be published, 1989.

E-16-603

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 December 1989 – 31 May 1990

3. TITLE OF PROPOSAL: Stability of Elastically Tailored Rotor Systems

4. CONTACT OR GRANT NUMBER: DAAL03-89-K-0007

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges and Lawrence W. Rehfield[1]

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   Hodges, Dewey H.; Atilgan, Ali R.; Fulton, Mark V.; and Rehfield, Lawrence W.: Free-Vibration Analysis of Composite Beams. Submitted to the *Journal of the American Helicopter Society.*

   Atilgan, Ali R.; and Rehfield, L. W.: Vibrations of Composite Thin-Walled Beams with Designed-in Elastic Couplings. Presented at the Fifth Japan-U.S. Conference on Composite Materials, Parthenon Tama, Tama-City, West of Tokyo, Japan, June 24 – 27, 1990.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   Principal Investigators:

   Dewey H. Hodges, Professor
   School of Aerospace Engineering
   Georgia Institute of Technology
   Atlanta, Georgia 30332-0150

   Lawrence W. Rehfield[1], Professor
   Dept. of Mechanical Engineering
   University of California at Davis
   Davis, California 95616

   Other Personnel:

   Mark V. Fulton, Graduate Research Assistant, School of Aerospace Engineering, Georgia Institute of Technology

BRIEF OUTLINE OF RESEARCH FINDINGS

A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to tailor the properties of composites to meet specific design requirements. Design of structures using material and geometric tailoring has received widespread attention during the last decade with the advent of composite materials. Tailoring of composite material properties has received relatively little attention, however, in the area of rotor blade design. The primary issue here is the overall suitability of tailored blades for practical applications. Blades must satisfy strength, weight, frequency, and lifetime requirements for structural stability; in addition, aeroelastic instabilities must lie outside the flight envelope.

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[1]Prof. L. W. Rehfield's effort in this project is supported under subcontract.

It is well known that kinematical couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize the couplings from the use of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. There also remains the possibility of tailoring the rotor blade structure and improving stability to the point that auxiliary mechanical dampers become unnecessary. Another possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. The principle is that the blade rotational speed, through extension-twist coupling, can be used to alter the twist of the blade to favorably influence the aerodynamics. These and other areas have strong potential for breakthroughs in design methodology.

The project has been underway for about a year and a half. During this time our analytical efforts have concentrated on achieving a better understand the design process and on the development of analyses that will predict the frequencies and mode shapes of rotating composite blades.

Design of Tailored Unbalanced Laminates

Rotor blades with either bending-twist coupling or extension-twist coupling designed in are most easily made with unbalanced angle plies (plies in only one off-axis direction). Until now, laminates of this type have been rarely used, as elastic couplings have been intentionally avoided. Consequently, very little is known of their damage and failure modes.

Practical guidance that is widely accepted in the aerospace industry is to place fibers in three non-parallel directions. The reasoning is that matrix-controlled damage is avoided by this practice. The most weight-efficient tailoring concepts lead to a combination of axial (0°) plies combined with an unbalanced configuration of angle plies. The (0°) plies carry the bulk of the centrifugal force, while the angle plies provide the elastic coupling, share a portion of the centrifugal loading and sustain the major share of torsional loads. In order to explore failure of these laminates, a computer code is under development which is based on the maximum strain failure criterion. A mix of (0°) and angle plies is considered, and failure under tensile loads is predicted. This work will help us better understand the practical use of composites in design of rotor blades.

Analysis of Free-Vibration Characteristics

The dynamic analysis may be efficiently performed as a two-stage operation. The first step is to determine the cross sectional stiffnesses by means of either a simple analytical technique or by a finite element method (a two-dimensional problem). The second step is a global bending-torsion-extension-shear analysis (a one-dimensional problem). The only part of results from the first step that we really need for the second step is the stiffness matrix. The results for the second step include the global dynamic behavior of the blade (such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics).

A mixed finite element method has been under development for use in a rotor stability analysis. In this method, the same very crude shape function is used for all unknowns (displacements, rotations, strain measures, stress resultants, generalized speeds, and momenta). The displacements and rotations converge with error inversely proportional to the number of elements squared. The stress resultants (sectional forces and moments), however, converge to the exact solution with error inversely proportional to the number of elements to the fourth power. This method allows for simple treatment of beams with variable geometry. At present, the method accounts for the elastic couplings as well as the geometric stiffness produced by rotation of the blade. The analysis method leads to relatively large dynamic matrices, including the Jacobian; all such matrices are very sparse, however, and *Mathematica* (a computerized symbolic manipulator) is used to obtain the Jacobian in closed form. Much of the code itself has been written by *Mathematica*. (We also attempted to factor the Jacobian with *Mathematica*, but this has not worked, because *Mathematica* was unable to factor a matrix this large in spite of its sparsity.) The code is presently capable of solving for the equilibrium deflections of a rotating blade. The sparsity of the Jacobian allows this operation to be done quite efficiently on a minicomputer. We are using the Harwell library sparse matrix subroutines. In the future, we will be adding linearized dynamics about the equilibrium and unsteady aerodynamics.

Free-Vibration Analysis of Composite Beams

Dewey H. Hodges*, Ali R. Atılgan, Mark V. Fulton†**
Georgia Institute of Technology, Atlanta, Georgia

and

Lawrence W. Rehfield ††
University of California, Davis, California

Abstract

Methods for predicting the natural frequencies and mode shapes of composite beams are presented. The sectional elastic constants are determined from two qualitatively different methods: simple analytical methods in which the stiffnesses are given in closed form and a detailed cross-sectional finite element method. The equations of motion are also solved in two ways: by an essentially exact integration method and by a mixed finite element method. All the methods are validated for the types of problems considered by comparison with previously published experimental data and numerical results. Predicted elastic constants based on the methods considered are presented, and differences between the results are discussed. Numerical results are also presented showing the effects of these differences on the predicted natural frequencies. It is shown that predicted free-vibration characteristics of composite beams can be sensitive to the assumptions used in determining the stiffnesses. Finally, the influence of ply layup on the natural frequencies and mode shapes is studied for thin-walled beams with circular cross sections.

Introduction

This paper addresses the calculation of natural frequencies and mode shapes for nonrotating composite beams. The purposes of the paper are (1) to compare the results of different methods for calculating the sectional elastic constants; (2) to present two methods

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Presented at the American Helicopter Society National Specialists' Meeting on Rotorcraft Dynamics, Ft. Worth, Texas, Nov. 13 – 14, 1989.

* Professor, School of Aerospace Engineering, Member, AHS.

** Post Doctoral Fellow, School of Aerospace Engineering, Member, AHS.

† Graduate Research Assistant, School of Aerospace Engineering, Member, AHS.

†† Professor and Division Leader, Aeronautical Science and Engineering, Member, AHS.

Presented at the Fifth Japan-U. S. Conference on Composite Materials
Parthenon Tama, Tama-City, West of Tokyo, Japan, 24 – 27 June, 1990.

Vibrations of Composite Thin-Walled Beams with Designed-in Elastic Couplings

Ali R. Atılgan*
School of Aerospace Engineering
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Atlanta, GA 30332-0150 USA

Lawrence W. Rehfield**
Department of Mechanical, Aeronautical, and Materials Engineering
University of California
Davis, CA 95616-5294 USA

ABSTRACT

Effects of elastic coupling mechanisms on the vibration behavior of thin-walled composite beams are evaluated analytically. The evaluation is accomplished for two mutually exclusive constructions, one designed for classical extension-twist coupling and the other for classical bending-twist coupling. Effects of nonclassical accompanying elastic couplings identified earlier by the authors are also explored. The predicted results indicate that the presence of classical and accompanying elastic couplings are necessary in order to model the vibration behavior of composite constructions. The predictions are validated by comparison with published experimental and numerical results.

INTRODUCTION

Composite material systems are currently primary candidates for aerospace structures. One key reason for this is the design flexibility that they offer. It is possible to tailor the structure to the application. A definition of elastic tailoring is the use of structural concept, fiber orientation, ply stacking sequence and a blend of materials to achieve specific performance goals. One important key to understanding the behavior of elastically tailored composite thin-walled beams is the analytical approach. Appropriate theories for analysis of thin-walled composite beams have been developed and validated by the authors and their collaborators (1, 2, 3, 4, 5, 6, 7). Agreement was found to be excellent in all of these cases. In this paper, vibration behavior of composite thin-walled beams are analytically explored. A survey and development of computational methodologies, including a transfer matrix method and a mixed finite element method, are presented in (7). Here it is intended to investigate some fundamental issues toward understanding the effects of elastic couplings on vibration behavior of composite thin-walled beams.

A THIN-WALLED COMPOSITE BEAM THEORY

Here attention will be directed to uniform, single cell beams. After the kinematics of the theory is summarized, we will then outline development of the equations of motion.

Kinematics

A thin-walled beam with closed, single-cell cross section is shown in Fig. 1. The coordinate direction x_1 is along a straight reference axis while x_2 and x_3 are the transverse coordinates of the cross section measured from the reference axis. Here the shear center of the cross section and x_1 axis projection have been assumed

* Post Doctoral Fellow. Member, ASC.
** Professor of Aeronautical Science and Engineering. Member, ASC, ASTM.

E-16-603

PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 June 1990 – 30 November 1990

3. TITLE OF PROPOSAL: Stability of Elastically Tailored Rotor Systems

4. CONTACT OR GRANT NUMBER: DAAL03-89-K-0007

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges and Lawrence W. Rehfield[1], Principal Investigators

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   Hodges, Dewey H.; Atilgan, Ali R.; Fulton, Mark V.; and Rehfield, Lawrence W.: Free-Vibration Analysis of Composite Beams. *J. American Helicopter Soc.*, to appear, 1991.

   Rehfield, Lawrence W.; and Atilgan, Ali R.: Understanding New Tailoring Mechanisms for Thin-Walled Composite Beams. *J. Composites Technology and Research*, submitted for publication, 1990.

   Atilgan, Ali R.; and Rehfield, Lawrence W.: Vibrations of Composite Thin-Walled Beams with Designed-in Elastic Couplings. *J. Composites Technology and Research*, submitted for publication, 1990.

   Rehfield, L. W.; Chang, Y. S.; and Atilgan, A. R.: New, Unusual and Nonclassical Behavior of Thin-Walled Composite Structures. To appear in the *Proceedings of the Eighth International Conference on Composite Materials*, Honolulu, Hawaii, July 15 – 19, 1991.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

   Profs. Dewey H. Hodges and Lawrence W. Rehfield[1]
   Dr. Ali R. Atilgan, Post Doctoral Fellow (partial support)
   Mark V. Fulton and Nafis A. Khan, Graduate Research Assistants

9. REPORT OF INVENTIONS (BY TITLE ONLY): none

BRIEF OUTLINE OF RESEARCH FINDINGS

Background: A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to tailor the properties of composites to meet specific design requirements. Design of structures using material and geometric tailoring has received widespread attention during the last decade with the advent of composite materials. Tailoring of composite material properties has received relatively little attention, however, in the area of rotor blade design. Blades must satisfy strength, weight, frequency, and lifetime requirements for structural stability; in addition, aeroelastic instabilities must lie outside the flight envelope. The primary issue here is the overall suitability of tailored blades for practical applications.

---

[1]Prof. L. W. Rehfield, Department of Mechanical, Aerospace, and Materials Engineering, University of California, Davis, California, was supported under subcontract.

It is well known that kinematical couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize the couplings from the use of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. There also remains the possibility of tailoring the rotor blade structure and improving stability to the point that auxiliary mechanical dampers become unnecessary. Another possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. The principle is that the blade rotational speed, through extension-twist coupling, can be used to alter the twist of the blade to favorably influence the aerodynamics. These and other areas have strong potential for breakthroughs in design methodology.

This project has been underway for two years. During this time our analytical efforts have concentrated on achieving a better understanding of the design process and on the development of analyses that will predict the aeroelastic stability of rotating composite blades. During this reporting period efforts were concentrated on the continued development of stability analyses for rotors with composite blades.

Overall Analytical Strategy: Aeroelastic stability analysis of composite blades must be performed as a two-stage operation. The first step is to determine blade cross-sectional stiffnesses by means of either an analytical technique or by the finite element method (a two-dimensional problem). The second step is to use these stiffnesses to conduct a dynamic bending-torsion-extension-shear aeroelasticity analysis (a one-dimensional problem). The results for the second step include the global dynamic behavior of the blade such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics.

Simpler Sectional and Stability Analyses: Extension of Rehfield's simplified thin-walled sectional analysis to include pretwist has begun. In conjuction with this, a conventional displacement modal stability analysis is under development which should serve as an independent check for results obtained from the mixed finite element analysis described in earlier progress reports and updated below. An independent check is needed because previously published analytical results for blades with extension-twist coupling lack the concomitant bending-shear coupling and are likely to be quite different from ours. We anticipate that with admissible functions based on Jacobi polynomials and quasi-steady, strip-theory aerodynamics, this model should run on a "PC" without difficulty. This latter effort began two months ago, and the code is about one-third complete.

Mixed Finite Element Stability Analysis: Our mixed finite element method has been under development for use in a rotor stability analysis, and is reaching maturity. In this method, very crude shape functions are used for displacements, rotations, strain measures, stress resultants, generalized speeds, and momenta; in spite of this, very accurate results are obtained. This method allows for simple treatment of beams with variable geometry and accounts for all possible elastic couplings. Because of the simple structure of the equations, *Mathematica* (a computerized symbolic manipulator) can be used to write the code.

The code is presently capable of solving for the equilibrium deflections of a rotating blade and the linear structural dynamics of small motions about equilibrium. This type of solution procedure is necessary for a stability analysis. Although the mixed method leads to relatively large matrices, their sparsity allows the solution to be done quite efficiently on a SUN 3/260. We are using the Harwell library sparse matrix subroutines to exploit the sparsity (which is of the order of 90%). The code has now been validated for both static large deflections and linear dynamics about equilibrium by correlation with experimental data for both isotopic and composite beams; the results are *excellent*. Yet to be validated features of the analysis include spanwise variable pretwist, mass and inertia properties, stiffnesses, distributed conservative forces, and distributed follower moments. In addition, aerodynamics based on Peters' dynamic inflow theory and corresponding lift, drag, and moment models are nearly ready for insertion into the code. Needed modifications to this formulation appear to be minimal and its addition to our code should not require much time. After the addition of these models, we will then be in a position to evaluate the aeroelastic stability of composite blades with all possible elastic couplings.

It should be noted that this mixed formulation is a candidate for use in the next major modification of 2GCHAS.

Submitted to the ASTM *Journal of Composites Technology and Research*, September, 1990

Understanding New Tailoring Mechanisms for Thin-Walled Composite Beams

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and

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Abstract

Two new elastic coupling mechanisms that occur in thin-walled composite tubular beams, bending-transverse shear and extension-transverse shear, are presented and evaluated. The evaluation is conducted for two limiting types of construction, one designed for classical bending-twist coupling and the other for extension-twist coupling. The new couplings naturally occur in a parasitic manner. They cause the beams to respond as if they are much more flexible for certain modes of deformation. The magnitude of the effects produced are enormous. The effectively increased flexibilities may hinder or limit the use of elastic coupling for structural tailoring.

Nomenclature

A	membrane stiffness matrix
A_e	enclosed cell area
C	beam stiffness matrix
c	circumference of cell wall
F	generalized force matrix
h	ply thickness
K	membrane stiffness matrix (for uniaxial and shear stress only)
L	beam length
M_x	twisting moment
M_y, M_z	bending moments

Key Words: Composite materials, thin-walled beams, coupling mechanisms, elastic tailoring

Presented at First USSR – US Symposium on Mechanics of Composite Materials, Riga, Latvia, USSR, May 23 – 25, 1989.

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Vibrations of Composite Thin-Walled Beams with Designed-in Elastic Couplings

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ABSTRACT

Effects of elastic coupling mechanisms on the vibration behavior of thin-walled composite beams are evaluated analytically. The evaluation is accomplished for two mutually exclusive constructions, one designed for classical extension-twist coupling and the other for classical bending-twist coupling. Effects of nonclassical accompanying elastic couplings identified earlier by the authors are also explored. The predicted results indicate that the presence of classical and accompanying elastic couplings are necessary in order to model the vibration behavior of composite constructions. The predictions are validated by comparison with published experimental and numerical results.

INTRODUCTION

Composite material systems are currently primary candidates for aerospace structures. One key reason for this is the design flexibility that they offer. It is possible to tailor the structure to the application. A definition of elastic tailoring is the use of structural concept, fiber orientation, ply stacking sequence and a blend of materials to achieve specific performance goals. One important key to understanding the behavior of elastically tailored composite thin-walled beams is the analytical approach. Appropriate theories for analysis of thin-walled composite beams have been developed and validated by the authors and their collaborators[1,2,3,4,5,6,7]. Agreement was found to be excellent in all of these cases. In this paper, vibration behavior of composite thin-walled beams are analytically explored. A survey and development of computational methodologies, including a transfer matrix method and a mixed finite element method, are presented in Ref. 7. Here it is intended to investigate some fundamental issues toward understanding the effects of elastic couplings on vibration behavior of composite thin-walled beams.

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NEW, UNUSUAL AND NONCLASSICAL BEHAVIOR OF THIN-WALLED COMPOSITE STRUCTURES*

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ABSTRACT

Recent attention has been given to the subject of elastic tailoring of thin-walled structures with composite materials. In the course producing desirable elastic couplings in thin-walled beams, two new coupling mechanisms have been found.[1,2] Also, nonclassical effects such as transverse shear deformations[3,4] and torsion-related warping[5] have been shown to be important even for thin-walled, slender beams.

Tailored response utilizing elastic coupling mechanisms for thin-walled tubular beams is achieved by skewing angle plies with respect to the beam axis. Two limiting archetype configurations are shown in Figs. 1 and 2. The circumferentially uniform stiffness (CUS) configuration produces extension-twist coupling with bending-transverse shear as a parasitic coupling mechanism. The other configuration, circumferentially asymmetric stiffness (CAS), produces bending-twist coupling with extension-transverse shear as a parasitic coupling mechanism. The angle θ in Figs. 1 and 2 denotes the dominant angle ply orientation in the upper and lower surface wall configurations.

The results of bending a CUS tube with a uniformly distributed loading (Fig. 3) appear in Fig. 4. Four predictions based upon the classical Bernoulli-Euler (BE) type of theory and three shear deformation theories are presented. The theory SD3 is the complete theory of Ref. 3 which has been thoroughly validated.[4-6] It is seen that the actual deflections (SD3) are approximately twice the classical predictions (BE). An analysis of these results indicates that bending-transverse shear coupling is the major cause of the beam appearing to be near twice as flexible as classical theory predicts.

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**Professor and Graduate Research Assistant, respectively.

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Another example of unusual behavior is provided by tailored wing concepts created at University of California, Davis. A central feature of the designs is the use of continuous filament grid stiffened configurations for the wing box covers. This type of stiffening concept is particularly useful for tailoring because unidirectional stiffeners can be oriented and placed to create a wide variation of elastic properties.

For one wing concept, a large effective Poisson ratio is desired for the wing covers. A balanced stiffener pattern is employed as shown in Figure 5. The effective Poisson ratio is A_{12}/A_{22}, where A_{12} and A_{22} are the membrane stiffnesses of laminate theory. Results for a graphite-epoxy cover are presented in Figure 6 for various degrees of stiffening. The parameter n is A_1/p_1h, where A_1 is the cross sectional area of the stiffener, p_1 is the pitch or distance between parallel rows of stiffeners and h is the skin thickness. A value of n of 1.5 is considered heavy stiffening. A Poisson ratio approaching three can be produced by a reasonable degree of stiffening.

These and other examples will be used to illustrate the new, unusual and nonclassical effects present in thin-walled composite structures.

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PROGRESS REPORT

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1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 December 1990 – 31 May 1991

3. TITLE OF PROPOSAL: Stability of Elastically Tailored Rotor Systems

4. CONTACT OR GRANT NUMBER: DAAL03-89-K-0007

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges and Lawrence W. Rehfield[1], Principal Investigators

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

   Hodges, Dewey H.; Atilgan, Ali R.; Fulton, Mark V.; and Rehfield, Lawrence W.: Free-Vibration Analysis of Composite Beams. *J. American Helicopter Soc.*, to appear, July 1991.

   Rehfield, L. W.; Chang, Y. S.; and Atilgan, A. R.: New, Unusual and Nonclassical Behavior of Thin-Walled Composite Structures. To appear in the *Proceedings of the Eighth International Conference on Composite Materials*, Honolulu, Hawaii, July 15 – 19, 1991.

   Hodges, Dewey H.; and Atilgan, Ali R.: Asymptotical Modeling of Initially Curved and Twisted Composite Rotor Blades, Paper no. 47, *Proceedings of the American Helicopter Society International Technical Specialists' Meeting on Rotorcraft Basic Research*, Atlanta, Georgia, March 25 – 27, 1991.

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: Profs. Dewey H. Hodges and Lawrence W. Rehfield[1], principal investigators; Dr. Ali R. Atilgan, Post Doctoral Fellow (partial support); Mark V. Fulton and Nafis A. Khan, Graduate Research Assistants

9. REPORT OF INVENTIONS (BY TITLE ONLY): none

BRIEF OUTLINE OF RESEARCH FINDINGS

Background: A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to tailor the properties of composites to meet specific design requirements. Blades must satisfy strength, weight, frequency, and lifetime requirements; and aeroelastic instabilities must lie outside the flight envelope. The primary issue is the overall suitability of tailored blades for practical applications. It is well known that elastic couplings can have a strong influence on rotor blade stability. Thus, any attempt to

[1]Prof. L. W. Rehfield, Department of Mechanical, Aerospace, and Materials Engineering, University of California, Davis, California, was supported under subcontract.

utilize the couplings of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. One possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. This and other uses have strong potential for breakthroughs in design methodology.

Aeroelastic stability analysis of composite blades must be performed as a two-stage operation. The first step is to *determine* blade cross-sectional stiffnesses, either by means of simplifying the analytical model so that the stiffnesses can be determined in closed form or by applying powerful dimensional reduction techniques which lead to a two-dimensional finite element solution for the stiffnesses. The second step is to *use* these stiffnesses to conduct a dynamic bending-torsion-extension-shear aeroelastic analysis of the blade. The results for the second step include the global dynamic behavior of the blade such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics.

The project is in its third year, and our efforts to date have concentrated on achieving a better understanding of the design process and on development of analyses that will predict the aeroelastic stability of rotating composite blades. During this reporting period we continued to develop and validate various aspects of the blade theories, especially for twisted blades, and stability analyses for rotors with composite blades.

Sectional Analyses: Extension of Rehfield's analytical thin-walled sectional methodology to include pretwist is still being worked on. There are three candidate sectional analyses: one is an improved kinematically-based analysis, one is totally new and flexibility-based, and the third is a sort of neo-classical theory which ignores direct transverse shear stiffness while maintaining the complex effects of the various elastic couplings. These theories are being evaluated for use in stability analyses. For use in elastic tailoring studies, we also have developed what we call the ideal tailored box model, which we intend to use in trend studies during the final period of this project. Moreover, in other ARO-sponsored work we have applied the variational-asymptotical method to initially curved and twisted composite beams, which leads to a sectional finite element solution for the stiffnesses. This theory is analogous to the neo-classical one above.

Simple Stability Analysis: The simplified stability analysis began during the last reporting period is still under development. It should serve as an independent check for the mixed finite element analysis described in earlier progress reports and updated below. The simplified analysis uses Jacobi polynomials as admissible functions and quasi-steady, strip-theory aerodynamics; the intent is that this model should run on a PC. This effort has been slower in coming than we originally hoped. The student working on this portion of the project devoted some time to preparing for the Ph.D. qualifying exam. Also, there were hardware problems with the Symbolic Computation Laboratory computers. Alternative strategies for this analysis are being considered at this time.

Mixed Finite Element Stability Analysis: Our stability analysis based on the mixed finite element method is near completion. It allows for simple treatment of beams with variable geometry and accounts for all possible elastic couplings. *Mathematica* (a computerized symbolic manipulator) has been used to write extensive portions of the code, which is presently capable of solving for the equilibrium deflections of a rotating blade and linearized dynamics of small motions about equilibrium. Both of these solutions are needed for stability analysis. The code has been validated for both static large deflections and linear dynamics about equilibrium by correlation with experimental data for both isotopic and composite beams; the results are *excellent*. An aerodynamic model has been adopted similar to that of GRASP (developed at the Aeroflightdynamics Directorate) and is being carefully validated and inserted into the code with the aid of *Mathematica*. After insertion of this model, we will then be in a position to evaluate the aeroelastic stability of composite blades with all possible elastic couplings. (The student working on this portion of the project completed his Ph.D. proposal during this reporting period.)

An unanticipated spinoff of this work has been the identification of a simple way to extend GRASP and 2GCHAS to work for composite rotor blades. This will be discussed with R. Ormiston this Summer at AFDD while Dr. Hodges is spending three weeks there. Researchers at SERI have also expressed an interest in this work for wind turbine blade modeling.

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PROGRESS REPORT

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1. ARO PROPOSAL NUMBER: 25327-EG

2. PERIOD COVERED BY REPORT: 1 June – 30 November 1991

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5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Dewey H. Hodges and Lawrence W. Rehfield[1], Principal Investigators

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

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8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD: Profs. Dewey H. Hodges and Lawrence W. Rehfield[1], principal investigators; Mark V. Fulton and Nafis A. Khan, Graduate Research Assistants

9. REPORT OF INVENTIONS (BY TITLE ONLY): none

BRIEF OUTLINE OF RESEARCH FINDINGS

Background: A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to tailor the properties of composites to meet specific design requirements. Blades

[1]Prof. L. W. Rehfield, Department of Mechanical, Aerospace, and Materials Engineering, University of California, Davis, California, was supported under subcontract.

must satisfy strength, weight, frequency, and lifetime requirements; and aeroelastic instabilities must lie outside the flight envelope. The primary issue is the overall suitability of tailored blades for practical applications. It is well known that elastic couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize the couplings of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. One possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. This and other uses have strong potential for breakthroughs in design methodology.

Aeroelastic stability analysis of composite blades must be performed as a two-stage operation. The first step is to *determine* blade cross-sectional stiffnesses, either by means of simplifying the analytical model so that the stiffnesses can be determined in closed form or by applying powerful dimensional reduction techniques which lead to a two-dimensional finite element solution for the stiffnesses. The second step is to *use* these stiffnesses to conduct an aeroelastic analysis of the blade obtaining measures of global dynamic behavior of the blade such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics.

The project is in its third year and concludes in about six months. Our efforts to date have concentrated on achieving a better understanding of the design process and on development of analyses that will predict the aeroelastic stability of rotating composite blades. During this reporting period we continued to develop and validate various aspects of the blade theories, especially for twisted blades, and stability analyses for rotors with composite blades.

Sectional Analyses: We have now settled on three sectional analyses in addition to NABSA which will be used and compared in the stability analysis: one is a new flexibility-based method for a closed, thin-walled cell developed recently developed by Rehfield; the second is also a closed, thin-walled cell analysis recently developed by Armanios, Berdichevsky, and Badir under other work being sponsored by ARO; the third is a finite element analysis based on asymptotical theory, developed by Hodges under other work being sponsored by ARO, which is able to account for the complex effects of the various elastic couplings without necessarily increasing the number of sectional coordinates beyond that of classical analyses. For use in elastic tailoring studies we also plan to use our "ideal tailored box" model. Descriptions of these modeling approaches in detail can be found in the European Rotorcraft Forum paper cited above and in papers which have been accepted for presentation at the upcoming AIAA SDM Conference.

Mixed Finite Element Stability Analysis: Our stability analysis based on a mixed beam finite element method is now completed and coded. It treats general composite beams with variable spanwise geometry and accounts for all possible elastic couplings in the six by six sectional stiffness matrix which it can accept from any source, finite element or analytical. The aerodynamic model is similar to that of GRASP (developed at AFDD) *Mathematica* (a computerized symbolic manipulator) has been used to write extensive portions of the code. The analysis is capable of solving for the equilibrium deflections of a rotating blade in air and linearized dynamic aeroelastic stability of small motions about the equilibrium configuration. The structural and structural dynamics parts of the code have been validated for non-rotating beams for both static large deflections and linear dynamics about equilibrium by correlation with experimental data for both isotopic and composite beams; the results are excellent. Presently the stability analysis is being validated based on previously published results for equilibrium deflections and stability eigenvalues. So far, the results are also excellent. After validation is satisfactorily completed, we plan to turn to systematic parameter studies for a variety of composite blades. If there is time, we plan to include some high-inflow cases appropriate for isolated blade stability analysis of tilt-rotor blades.

Technology Transfer: Dr. Hodges spent 3 weeks at AFDD working on 2GCHAS technical issues. There seems to be considerable interest at AFDD for using our sectional modeling in 2GCHAS. On the other hand, mixed finite element methods for rotor blade stability analysis may have a long way to go to achieve the acceptance achieved by more conventional methods. The reasons for this stem from uncertainties related to the numerical stability of time domain analyses based on mixed methods.

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Stability of Elastically Tailored Rotor Systems

Dewey H. Hodges, Lawrence W. Rehfield, and Mark V. Fulton

Final Technical Report

1 December 1988 – 30 September 1992

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Research Supported by the U.S. Army Research Office

Contract DAAL03-89-K-0007 (ARO Proposal Number: 25327-EG)

Principal Investigators: Dewey H. Hodges and Lawrence W. Rehfield[1]

Grant Monitor: Dr. Gary L. Anderson

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[1]Presently with the University of California at Davis

Introduction

Background

A significant attribute of laminated composites is their design flexibility. The layers or plies of a laminate are, in fact, modular units which can be selected to provide distinct material properties and fiber orientations. It is possible, therefore, to tailor the properties of composites to meet specific design requirements. Relatively little work has been done to apply principles of aeroelastic tailoring to rotor blades. Rotor blades must satisfy strength, weight, frequency, and lifetime requirements; and aeroelastic instabilities must lie outside the flight envelope. The primary issue when it comes to design of composite rotor blades is the overall suitability of tailored blades for practical applications.

It is well known that elastic couplings can have a strong influence on rotor blade stability. Thus, any attempt to utilize the couplings of composite materials in rotor blade design must include a concomitant investigation of the influence of the coupling on stability. One possibility for utilizing tailoring in rotor system design is based upon the creative use of extension-twist coupling to achieve enhanced performance. This and other uses have strong potential for breakthroughs in design methodology.

Objectives

In this work our objectives included (1) the development of models and analysis methodology for rotor stability in hover, incorporating all relevant nonclassical structural effects; (2) the validation of the models and analysis methodology, including convergence; and (3) the conduct of limited design studies with the tools developed in order to enhance understanding of the role of elastic couplings on stability. We believe that all these objectives were met. What follows is a brief summary description of the results of our work. Details may be found in the published papers listed at the end of the report.

Unique Features of Work

New Models and Analysis Methodology The aeroelastic stability analysis of composite blades should be performed as a two-stage operation for the best combination of computational efficiency and accuracy. The first step is to *determine* blade cross-sectional stiffnesses. This can be done either by means of simplifying the analytical model so that the stiffnesses can be determined in closed form or by applying powerful dimensional reduction techniques to the three-dimensional representation, which leads to a two-dimensional finite element solution for the stiffnesses and a corresponding set of one-dimensional (global or beam) equations. The second step is to *use* the sectional stiffnesses and the corresponding beam equations to conduct an aeroelastic analysis of the rotor blade, obtaining measures of the global dynamic behavior of the blade such as frequencies and mode shapes without aerodynamics or stability eigenvalues with aerodynamics. An optional third step is to substitute the results from the global analysis back into approximate three-dimensional recovery relations to get pointwise asymptotically correct approximations of displacement, strain, and stress. For simplicity, this third step was not undertaken in our work.

In the course of our work several sectional analysis methods were developed, adopted, and studied. Analyses developed include one based on potential energy and another based on complementary energy by Dr. Rehfield. Codes that were adopted and studied include Marco Borri's code NABSA, based on Giavotto et al. (1983); Mark Nixon's code TAIL based on

Rehfield (1985); Carlos Cesnik's code VABS, based on the work of Hodges et al. (1992); and a code based on the work of Badir (1992). These codes were used because they have been shown to be accurate and they were available at the time we needed them.

The global beam equations of motion are those of Hodges (1990) and are solved directly from the mixed variational statement developed therein. Our stability analysis was formulated from this variational statement with simple mixed finite element shape functions. The finite element shape functions are the crudest possible based on the variational statement which is in the "weakest possible form." For example, all the unknown field variables are represented as piecewise constant within the element. This allows for exact element quadrature by inspection. The resulting coefficient matrices are extremely sparse, and taking advantage of that sparsity leads to a very efficient computational scheme for the nonlinear equlilibrium solution, which runs on workstations in a few minutes.

The resulting aeroelastic stability analysis code, STAB, accounts for all possible material elastic couplings and uses an aerodynamic model similar to that of GRASP (developed at the Aeroflightdynamics Directorate). *Mathematica* (a computerized symbolic manipulator) was used to write extensive portions of the code. STAB is capable of solving for the equilibrium deflections of a rotating blade and linearized dynamics of small motions about equilibrium for the hovering flight condition. Both of these solutions are needed for stability analysis.

Validation Studies The analyses developed under this grant were thoroughly validated. The types of validation include linear and nonlinear static deflection of isotropic and composite beams, free-vibration frequencies and modes for nonrotating isotropic and composite beams, and modal damping for hingeless rotors made of isotropic and composite materials. While not exhaustive, these sorts of validation studies ensure that all structural effects are represented accurately. These validation studies were done in an incremental fashion in order to gain confidence in each part of the analysis and code. In every case our results are at least as good as published results, and in most cases they are noticeably better. In validation of the large-deformation characteristics of composite beams, for example, we were able to calculate stiffness properties from "first principles" producing deflections which agreed with experiment much better than existing theories. We also validated our finite element code against our transfer matrix code. Note that the transfer matrix method is capable of calculating the frequency of each mode with comparable accuracy, whereas the finite element results deteriorate as the number of the mode increases. Thus, if the discussion is restricted to the lower-frequency modes, the finite element results can be used with confidence. As part of the validation, we also determined the convergence properties of the mixed finite element method. It was found that 4 to 8 elements were sufficient for static analysis, while 16 were sufficient for stability analysis. In our study of the modes and frequencies of nonrotating beams we found that for three modes of any one type (bending or torsion, say) to be within 2% requires 32 elements.

Concerning stiffness and the design process, analysts would generally prefer to work on raw material and geometric properties whenever possible. For instance, summing ply thicknesses and presuming that each ply retains its nominal engineering properties makes life simple. We note that this simplicity might produce unrealistic results. Indeed, a trend described by Minguet and Dugundji was confirmed and studied further in our work – namely, laminate thickness measurements taken in two different stages of the curing process produce different stiffnesses, and physical reasoning leads to a still different thickness and corresponding stiffnesses. The last thickness is the hardest to obtain in the design process, but it leads to the most accurate stiffnesses.

There is very little in the literature that can aid in validation of predictive methods for natural frequencies of vibration for anisotropic beams of solid cross section. One study, Abarcar and Cunniff (1972), does contain experimental results with which results from the present work were compared. Our purely theoretical results for the free-vibration characteristics agreed with experimental data very well –.better, in fact, than some of the theoretical results published by the experimenters based on identification of the beam properties from experiment!

Results from STAB also agree very closely with the U.S. Army code PFLT for aeroelastic stability of isotropic blades in hover. We believe that STAB does a better job of calculating the larger deflections than does PFLT. Indeed, some deficiencies in the PFLT results turned out to be due to its reliance on an ordering scheme. Because STAB does not use an ordering scheme, it does not exhibit these problems.

For composite blades, STAB's correlation with Yuan et al. (1992) is quite good, but its correlation with Hong and Chopra (1984, 1985) is quite poor except at zero ply-angle. Differences seem to be both quantitative and qualitative. The qualitative differences are especially apparent since their cases II and IV are said to exhibit a flap divergence for $C_T/\sigma = 0.10$ in Hong and Chopra (1984, 1985). This divergence, however, did *not* appear when generating the current results. In these cases their trim algorithm diverged, which may have been due to a combination of modeling errors in the work of Hong and Chopra plus their reliance on an ordering scheme. Also, we note that the excellent agreement between the analysis of Yuan et al. (1992) and ours, along with the observation of Fulton (1992) that the large variations in the damping due to modifications only affecting a small fraction of the cross section, are strong indications that the present model is more likely to be correct on this point.

All essential nonclassical effects were included in our model; indeed, the 3-D representation from which the model is derived accounts for all possible deformation of the blades and all possible elastic couplings. The approximations are in the development of the 1-D constitutive model, which cannot retain all of the 3-D information exactly. The nature of the asymptotical approximation is that certain short wave-length phenomena, such as the restrained warping effect, are not represented.

Modeling and Analysis Development

A common design for rotor blade sections is to utilize thin-walled construction. This is both weight-efficient and in accordance with the frequently-used approaches to manufacture with composite materials. The code TAIL developed by Robert Hodges and Mark Nixon, based upon the early work of Rehfield (1985), applies to thin-walled single cell blade box configurations. The thin-walled section, besides its practical usefulness, is a case where closed-form integral expressions for section stiffness are available. From a design and manufacturing point of view, the single-cell configuration is likely to be preferred for creating aeroelastically tailored configurations with elastic coupling incorporated, perhaps by layup over a honey comb shaped core that serves as a male tool for ease of manufacture. Consequently, special efforts have been made to improve the modeling of these configurations.

Nearly all modeling accomplished to date has been based upon displacement formulations and potential energy considerations. It is well known that such approaches yield analysis results that *overestimate stiffness*. This is unconservative in stability studies. Consequently, Dr. Rehfield developed a new modeling approach base upon complimentary energy considerations which tends to *underestimate stiffness*. This new theoretical approach yielded two benefits. The first is that compliances or flexibilities (the inverse of stiffnesses) are determined directly in terms of closed-form integrals. Secondly, the out-of-plane warping displacement can be estimated from closed-form expressions in a manner consistent with the thin-walled approximation. This formulation was presented to ARO in an informal report (Rehfield 1991a).

The original work which uncovered a new range of elastic coupling types (Rehfield 1985) was a linear, small-deflection theory. This has been extended to the geometrically nonlinear range of moderate rotations. While not as general as Dr. Hodges' formulations, this approximation yields an extremely useful means of evaluating the geometric stiffness effects of centrifugal blade axial loads on torsional stiffness and yields consistent static stability equations (Rehfield 1991b). Some static stability (i.e., buckling) results will be presented in a forthcoming invited paper (Rehfield 1993).

With the out-of-plane warping displacement known from complimentary energy-based theory above, it may be used to create a theory for beams with initial twist which is, relatively speaking, simple and easy to use. It was presented also to ARO in an informal report (Rehfield 1991c).

Limitations of time, resources and Dr. Rehfield's major health problems prevented the integration of these works (Rehfield 1991a, 1991b, 1991c) into design studies. Furthermore, considerable time and effort must be devoted to create sufficient understanding to permit rational design in the presence of both elastic coupling and pretwist.

Together with the above modeling issues which are "analysis" oriented, a preliminary design model for cross sections has been developed which aids in defining coupled configurations in the design process. We call it the "Ideal Tailored Box Model." This is a model specifically designed to be used in elastic tailoring studies of rotor blades and high aspect ratio wings. It is very simple and serves as a closed-cell counterpart to the "Ideal I-Beam Model" for bending about a single axis. It should prove very useful for (1) teaching, (2) optimization and parametric studies, (3) facilitating physical understanding, (4) providing Qualitative trend information and (5) isolating independent designer-controlled mechanisms. It's development was jointly supported by NASA, and it is fully described in (Rehfield, Chang and Zischka 1992). Also given is an application to a rotor blade configuration supplied to us by Mark Nixon. This section model has the added advantage that all stiffness and compliances are found from simple, closed-form expressions.

Design Information

A survey of companies, which included Bell, Boeing, McDonnell Douglas and Sikorsky, was performed to learn about blade structural design approaches used in industry. What has emerged is a picture of individual company approaches which depend upon company historical data from previous designs, proprietary practices, iteration, and static and fatigue coupon testing. It has not been possible to unify the different approaches. This is made difficult, in part, by the ways that the individual companies organize their work units and by their use of company proprietary software that is often poorly documented and has undergone evolutionary modifications over time.

Reliance upon historical data and flight test data for design insures that innovation will likely be slow slow in coming. Parameter ranges that depart far from previous experience will be resisted. In particular, balanced layups which exhibit no elastic coupling are used to reduce or eliminate manufacturing-related warping of parts.

We believe that unbalanced layups are the most efficient for producing elastic coupling. Relatively little is known about failures or damage processes in such configurations. Some preliminary tests run at Northrop[2] suggest that a "scissor-type" damage mode can occur. In the absence of more information and data, we have dealt with configurations which are similar to those treated previously in the literature. This has the additional benefit of validating the analytical results.

Design Studies

Two soft in-plane rotor configurations, R1 and R2, were used for aeroelastic stability design studies. Rotor R2 was an extension-twist coupled rotor based upon the rotor of Hodges et al. (1987). Rotor R1, however, had two laminate designs, $L2_e$ ($[0°_2/\zeta_4]$) and $L3_e$ ($[\zeta,\zeta-90°,\zeta,(\zeta-90°)_2,\zeta]$), both of which were extension-twist designs.

Although the laminate for rotor R2 was fixed, the laminates for rotor R1 had a variable ply angle. The R1 laminate designs, along with their accompanying box beam dimensions (which were constrained to fit within the selected airfoil), were chosen because they gave realistic nondimensional rotating frequencies which did not significantly vary as the ply angle was swept from zero to ninty degrees. In addition, the two laminate designs were considered to fairly represent the entire class of extension-twist designs.

For rotor R2, a sweep of the thrust level was used to investigate the lead-lag damping variation with hovering flight condition. Results were calculated for various stiffness models. It was demonstrated that a significant error appeared for this case (especially at high thrust levels) when bending-shear coupling was neglected.

For rotor R1, sweeps of thrust level were made for each laminate design for various values of the ply angle. These studies demonstrated, as suggested by Hong and Chopra (1984, 1985), that the elastic coupling available from laminate design was able to noticeably affect the lead-lag damping level for most thrust levels. In general, however, these laminate designs were not very sensitive to either the direct shear effects or the nonclassical (parasitic) coupling of bending-shear.

In general, the accuracy of the composite predictions was found to depend on the quality of the cross-sectional stiffnesses. The two-dimensional analyses used, however, gave nearly identical results for many cases because of their high quality. The performance of the "classical" stiffnesses (which ignore shear deformation effects), however, was poor for several cases.

Recommendations

In spite of the generality of the work done under this grant, there are certainly areas which deserve additional work. Indeed, it should be emphasized that far more design studies should be done than those we were able to complete once our model was validated. Even so, the model itself could be made more applicable to realistic helicopter design by including "better" aerodynamics, such as the Peters-He model and forward flight effects, and by also modeling the flexbeam or yoke (a more complete rotor system model). We also strongly favor the creation of additional experimental data sets for validation and correlation of composite blade analyses.

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[2]Deo, R., private communication to Dr. Rehfield, Northrop Corporation, Hawthorne, California, 1991.

It is clear that we need to emphasize "synthesis" now that many "analysis" issues are resolved by this work and others. Additional understanding of elastic couplings on dynamics is still needed as are guidelines for design of elastically coupled rotors.

Contributions

Thesis

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[3]Not formally published due to Dr. Rehfield's health problems.

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